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Janssen, Leon P.B.M.; Mościcki, Leszek

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## 15

### Scale-Up of Extrusion-Cooking in Single-Screw Extruders

*Leon P.B.M. Janssen and Leszek Mościcki*

#### 15.1

##### Introduction

Scale-up rules provide the possibility to transfer knowledge obtained on small scale laboratory equipment to large scale production units. The principle of scale-up is that equations describing the behavior of process equipment can be written in a dimensionless form. If the resulting dimensionless groups are kept equal in the small scale and in the large scale equipment, the solutions of the various equations remain constant in a dimensionless form. In extrusion cooking the significance of scale-up is twofold: it provides the possibility for product and process development on a small scale before doing the final trials on the production equipment and it gives the possibility to have a smooth transition to new equipment if a significant increase in production is needed.

Scale-up of extrusion-cooking suffers from the same general problems that are encountered in many other processes in the process industry:

- on scaling up, the surface to volume ratio decreases and therefore the possibilities for heat transfer are limited in large scale equipment,
- at equal temperature differences the temperature gradients, and therefore the heat fluxes, are smaller in large scale equipment,
- at equal shear fields in large scale and small scale equipment diffusion limitations connected to distributive mixing can be more predominant in large food extruders.

Various theories on the scale-up of single-screw food extruders exist. Due to the high viscosities a considerable amount of the process energy is transferred into heat by viscous dissipation. Therefore, the thermal considerations will dominate the scale-up rules and an important aspect is the extent to which the process is adiabatic or not. If the process can be considered to occur adiabatically a sufficient condition for scale-up will be that the energy input per unit throughput is constant and the average temperature of the end product will be the same in the small scale and the large scale

equipment. If this is not the case, similar temperature profiles in both types of equipment, called complete thermal similarity, are required.

The degree to which the process is adiabatic can be estimated from the Brinkmann number, that can be rewritten for extruders as [2, 12]:

$$Br = \frac{\mu v^2}{\lambda \Delta T} = \frac{\mu(\pi ND)^2}{\lambda \Delta T} \quad (15.1)$$

where  $\lambda$  is the thermal conductivity of the starch mass ( $\text{W m}^{-2} \text{K}^{-1}$ ) and  $\Delta T$  is the temperature difference between the mass and the barrel wall. If this Brinkmann number is much larger than unity, adiabatic scale-up is acceptable.

A particular dependence is the quadratic occurrence of the diameter. This implies that the Brinkmann number is generally large for production machines. It is generally not possible to keep the Brinkmann number constant for large scale and small scale machines. To obtain reliable predictions on small scale machines the Brinkmann number for this machine should at least be much larger than unity, which set its limitations to the minimum screw diameter of the small scale machine. If this number is smaller than unity for laboratory machines reliable scale-up is not possible.

In order to obtain complete thermal similarity, the screw rotation rate has to decrease drastically with increasing screw diameter, as compared to the adiabatic case [7, 9]. As a result, the scale factor for the throughput is only 1.5 for Newtonian fluids (and decreases even further for fluids with pseudo-plastic behavior). This scale-up factor ( $q$ ) for the throughput is defined from:

$$\left[ \frac{Q}{Q_0} \right] = \left[ \frac{D}{D_0} \right]^q \quad (15.2)$$

where  $Q$  denotes the throughput,  $D$  the screw diameter and the subscript 0 indicates the small extruder. In the case of adiabatic scale-up a scale-up factor up to 3 can be achieved. For a standard industrial extruder series in a first approximation it may be stated that:

$$\left[ \frac{Q}{Q_0} \right] = \left[ \frac{D}{D_0} \right]^{2.8}$$

When an extruder is scaled it is important to keep the process in the large machine as much as possible similar to that in the small machine. Complete similarity is often not possible or is impractical and choices in similarity have to be made. Several types of similarities can play a role in the scale-up of an extruder:

- *Geometric similarity* exists if the ratio between any two length parameters in the large scale equipment is the same as the ratio between the corresponding lengths in the small scale model. This is not necessarily the case, as will be seen later, but in general this condition can be very convenient.
- *For hydrodynamic similarity* two requirements should be fulfilled: The dimensionless flow profiles should be equal and for twin screw extruders, both extruders

should have the same (dimensionless) filled length. Equal dimensionless flow profiles lead to equal shear rates in corresponding locations, but not to equal velocities.

- *Similarity in residence times* means equal residence times in the small scale and large scale equipment. This is a requirement that is often not fulfilled in extrusion processes and in thermoplastic starch extrusion this can only be realized if the scale-up is adiabatic.
- Absolute *thermal similarity* is difficult to achieve, as stated before. This similarity indicates equal temperatures in all corresponding locations. A distinction has to be made between processes with small heat effects and those with high heat effects. For adiabatic processes where the heat generation is far more important than heat removal to the wall, similarity based on over-all energy balances is generally used. Although, strictly speaking, this does not lead to thermal similarity, equal average end temperatures of the product lead to far more favorable scale-up rules.

## 15.2

### Basic Analysis

To derive rules for scale-up, all parameters are assumed to be related to the diameter ratio by a power-relation. For this purpose, in this chapter all basic parameters will be written in capitals, whereas the scale-up factors will be written in small print. This implies that all relevant parameters can be related to the screw diameter according to:

$$\begin{aligned} \left[ \frac{N}{N_0} \right] &= \left[ \frac{D}{D_0} \right]^n; \left[ \frac{P}{P_0} \right] = \left[ \frac{D}{D_0} \right]^p; \left[ \frac{\mu}{\mu_0} \right] = \left[ \frac{D}{D_0} \right]^v; \\ \left[ \frac{H}{H_0} \right] &= \left[ \frac{D}{D_0} \right]^h; \left[ \frac{L}{L_0} \right] = \left[ \frac{D}{D_0} \right]^l; \left[ \frac{\tau}{\tau_0} \right] = \left[ \frac{D}{D_0} \right]^t; \left[ \frac{R}{R_0} \right] = \left[ \frac{D}{D_0} \right]^r \end{aligned} \quad (15.3)$$

where  $N$  is the rotation rate of the screws,  $P$  the die pressure, and  $\mu$  the viscosity. The back flow  $Q_b$  is an important parameter in scaling up. For closely intermeshing twin screw extruders it signifies the total amount of leakage flows, for single screw, self-wiping and non-intermeshing extruders it signifies the pressure flow.  $H$  denotes the channel depth,  $L$  the screw length,  $\tau$  the residence time in the extruder and  $R$  the pumping efficiency [1, 3, 10].

For thermal scale up rules two more parameters have to be used, the Greaz number ( $Gz$ ) that will be defined later and the Brinkmann number ( $Br$ ). These dimensionless numbers follow from writing the energy balance in a dimensionless form and the scale up notation for these groups reads:

$$\left[ \frac{Gz}{Gz_0} \right] = \left[ \frac{D}{D_0} \right]^{gz} \quad \text{and} \quad \left[ \frac{Br}{Br_0} \right] = \left[ \frac{D}{D_0} \right]^{br} \quad (15.4)$$

### 15.3

#### Summary of Equations Used

Scale-up rules are necessarily rather mathematical in nature. In this paragraph the extruder equations used are summarized.

The throughput of a single-screw extruder can be written as:

$$Q = \frac{1}{2} \pi^2 N D^2 H (1-a) \sin \theta \cos \theta \quad (15.5)$$

where  $\theta$  is the flight angle and  $a$  is the throttle coefficient:

$$a = \frac{H^2 \Delta P \tan \theta}{6 \mu (\pi N D) L} \quad (15.6)$$

The equation for the motor power in the pump zone can be written as:

$$E = \frac{(\pi N D)^2 W L}{H \sin \theta} (\cos^2 \theta + 4 \sin^2 \theta + 3a \cos^2 \theta) \quad (15.7)$$

where  $W$  is a channel width.

For use in scaling rules this equation can be simplified for screws with the same flight angle to:

$$E = \text{const} * \frac{\mu D^3 N^2 L}{H} \quad (15.8)$$

The pumping efficiency of the extruder is the ratio of the energy used for pumping the material and the total energy input into the extruder [1, 2, 7, 9].

$$R = \frac{QP}{E} \quad (15.9)$$

Thermal similarity yields from the energy balances:

$$\rho C_p \left( \frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + q \quad (15.10)$$

In this equation  $q$  is the heat produced by viscous dissipation:

$$q = 2\mu \left\{ \left( \frac{\partial v_x}{\partial x} \right)^2 + \left( \frac{\partial v_y}{\partial y} \right)^2 + \left( \frac{\partial v_z}{\partial z} \right)^2 \right\} + \mu \left\{ \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)^2 + \left( \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right)^2 + \left( \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right)^2 \right\} \quad (15.11)$$

If the equations above are made dimensionless there remain two important dimensionless numbers that govern the heat balances in the extruder, the Graetz number and the Brinkmann number.

$$Gz = \frac{UH^2}{aL} \quad \text{and} \quad Br = \frac{\mu U^2}{\lambda \Delta T} \quad (15.12)$$

where  $\lambda$  is thermal conductivity,  $T$  the temperature,  $U = \pi N D$ .

The Graetz number accounts for the development of the temperature profile, while the Brinkmann number signifies the ratio between viscous dissipation and heat conduction to the wall.

## 15.4

### Kinematic Similarity

Kinematic similarity means equal shear levels in the small and the large extruder [7]. Its importance is coupled to the requirements for:

- equal mixing in small and large machines,
- equal distribution of viscous dissipation,
- equal influence of non-Newtonian rheological effects.

For the throughput of the small laboratory extruder we can write:

$$Q_0 = \frac{1}{2} \pi^2 N_0 D_0^2 H_0 (1-a_0) \sin \theta_0 \cos \theta_0 \quad (15.13)$$

and for the throughput of the production machine:

$$Q = \frac{1}{2} \pi^2 N D^2 H (1-a) \sin \theta \cos \theta \quad (15.14)$$

If the screws of the small and the large machine have the same screw angle, which is the same as the same dimensionless pitch we may write:

$$\frac{Q}{Q_0} = \frac{N D^2 H}{N_0 D_0^2 H_0} \frac{(1-a)}{(1-a_0)} \quad (15.15)$$

and if we process both machines with the same throttle coefficient:

$$\frac{Q}{Q_0} = \frac{N}{N_0} \left( \frac{D}{D_0} \right)^2 \frac{H}{H_0} \quad (15.16)$$

Introducing the diameter ratios as defined before:

$$\left( \frac{D}{D_0} \right)^q = \left( \frac{D}{D_0} \right)^n \left( \frac{D}{D_0} \right)^2 \left( \frac{D}{D_0} \right)^h = \left( \frac{D}{D_0} \right)^{n+2+h} \quad (15.17)$$

gives the exponent equation:

$$q = n + 2 + h \quad (15.18)$$

Because both machines operate with the same throttle coefficient:

$$a = a_0 \rightarrow \frac{H^2 \Delta P}{6\mu(\pi N D) L} \tan \theta = \frac{H_0^2 \Delta P_0}{6\mu_0(\pi N_0 D_0) L_0} \tan \theta_0 \quad (15.19)$$

and equal throttle coefficients leads to:

$$2h + p - v - 1 - n - \ell = 0 \quad (15.20)$$

For equal velocity gradients an extra equation is necessary:

$$\frac{\pi ND}{H} = \text{constant}$$

and therefore:

$$h = n + 1 \quad (15.21)$$

for kinematic similarity both Equations (15.19) and (15.20) must be valid:

$$p = \ell - h + v \quad (15.22)$$

These results have to be combined with geometrical considerations or with thermal scaling rules.

## 15.5

### Geometrical and Kinetic Similarity

Geometrical similarity is often used for its simplicity but it is not a strong requirement. Especially in processing vegetable raw materials, where temperature and temperature homogeneity are very important, the principle of geometric similarity of small and large scale equipment cannot always be retained. Geometric similarity means that all dimensions scale in the same way, or:

$$l = 1 \quad \text{and} \quad h = 1 \quad (15.23)$$

Geometric and kinematic similarity follow from a combination of this equation with Equations 15.18, 15.21 and 15.22 resulting in

$$n = 0; \quad q = 3 \quad \text{and} \quad p = v \quad (15.24)$$

This means that if we scale our extrusion cooking process in a single-screw extruder and the temperature profiles are of lesser importance:

- the rotation speed must remain the same
- the throughput should be kept proportional to  $D^3$
- the die should be designed such that the pressure ratio equals the ratio between the end viscosities.

In this case no consideration is given to the temperature development.

## 15.6

### Motor Power and Torque

The motor power in the extruder can be approximated to:

$$E = \text{const} * \frac{\mu D^2 N^2 L}{H} \quad (15.25)$$

It should be realised that this equation does not comprise the power needed to transport the solid bed, however, this is not important for the thermal considerations in the next paragraphs.

The scale factor of the motor power can be defined as:

$$\frac{E}{E_0} = \left( \frac{D}{D_0} \right)^e \quad (15.26)$$

and we find:

$$e = 3 + 2n + \ell + v - h \quad (15.27)$$

and for the torque:

$$m = 3 + n + \ell + v - h \quad (15.28)$$

## 15.7

### Equal Average End Temperature

Two types of thermal similarities can be used: equal average end temperatures and similar temperature profiles [5, 6, 12]. The concept of equal average end temperatures can be applied if the extruder operates adiabatically or if  $Br \gg 1$ . In this case scaling up has to proceed according to equal motor power per unit throughput:

$$\frac{E}{Q} = \text{const} \quad \text{or} \quad e - q = 0 \quad (15.29)$$

With Equation 15.20 this leads, for equal viscosities and die pressures ( $v = 0$  and  $p = 0$ ), to

$$2h = 1 + n + l \quad (15.30)$$

In this case various degrees of freedom are still retained.

## 15.8

### Similar Temperature Profiles

From the dimensionless energy equation it follows that thermal similarity can be attained if the dimensionless numbers of Graetz and Brinkmann are the same for both sizes of machines.

Because:

$$Br = \frac{\mu(\pi ND)^2}{\lambda \Delta T} \quad (15.31)$$

we find for materials with the same heat conductivity ( $\lambda$ ) that thermal similarity is attained if:

$$v + 2n + 2 = 0 \quad (15.32)$$

This means that for materials with the same viscosity:  $n = -1$ .



From:

$$Gz = \frac{\pi NDH^2}{aL} \quad (15.33)$$

follows at equal heat diffusivity  $a$ :

$$1 + n + 2h - \ell = 0 \quad (15.34)$$

leading to thermal similarity (equal  $Br$  and  $Gz$  numbers) if:

$$2h = \ell \quad (15.35)$$

For extruders with equal length to diameter ratios ( $\ell = 1$ ) the channel depth must decrease according to  $h = 1/2$  which gives together with Equation 15.18:

$$q = 2 + n + h = 1.5 \quad (15.36)$$

or:

$$\frac{Q}{Q_0} = \left[ \frac{D}{D_0} \right]^{1.5} \quad (15.37)$$

From an economical point of view this is very unfavorable, and should only be applied in very special situations.

## 15.9

### Similarity in Residence Times

Equal residence time can be achieved if the volume divided by the throughput remains constant, or, if we define  $Z$  as the average residence time [4, 8]:

$$Z = \text{const} \frac{HLW}{Q} \quad (15.38)$$

which yields for screws with equal helix angle:

$$z = h + 1 + l - q \quad (15.39)$$

or with:

$$q = 2 + n + h \quad (15.40)$$

we find that:

$$z = l - n - 1 \quad (15.41)$$

For screws with geometric similarity, this means that ( $l = -1$ ,  $h = 1$  and  $z = -n$ ), equal residence times are only possible if the rotation speed is constant. In other cases equal residence times can only be obtained by changing the screw length, according to:

$$l = 1 + n \quad (15.42)$$

**Table 15.1** Application of Equation 15.30, giving a variety of possibilities for scaling rules.

$n$	$h$	$q$
-1.0	0.5	1.5
-0.6	0.7	2.1
-0.4	0.8	2.4
0.0	1.0	3.0

## 15.10

### Guidelines for Scaling

In extrusion-cooking generally both heat of conduction and heat of dissipation are important in the process. In small machines the Brinkmann number is relatively small but in larger machines the dissipation becomes more dominant and the process becomes more adiabatic. Because the thermal problems are predominant the basis for the guide lines are Equation 15.30, this equation can be combined with various other (less strict) requirements. Application of Equation 15.30 gives a variety of possibilities for scaling rules. The results for screws with equal length to diameter ratio, for instance, are shown in Table 15.1).

Equal end temperatures with adiabatic operation still leave the degrees of freedom to scale according to similar temperature profiles (of course!) with  $q = 1.5$  or to scale kinematically with  $q = 3$  and with values in between. For the design of extrusion-cookers this means that the thermal stability of the material and of the process are important. It can be envisioned that, for the compounding process for the preparation of starch, kinematic scale-up is preferred because temperature effects are still mildly important but kinematic similarity is important to obtain the same mixing mechanism (and therefore the same material) in the small and large scale process [8, 11]. On the other hand, for processes like extrusion of snack pellets, thermal similarity is extremely important, leading to thermal scale-up. Profile extrusion and sheet extrusion are “in between” processes and could be designed with  $n = -0.4$  and  $h = 0.8$  leading to  $q = 2.4$ .

In the examples above the  $L/D$  ratio remains constant but the screw length can also be changed to retain extra degrees of freedom. This leads to a three-dimensional matrix of parameters, but is outside the scope of this book.

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